END

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

②

AD-A144 300

ALTERNATIVE LABELLING SCHEMES IN ELECTROMAGNETIC TOPOLOGY

R.S. Noss

LuTech, Inc
3516 Breakwater Court
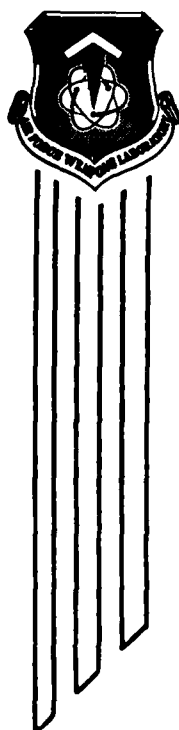Hayward CA 94545

June 1984

Final Report

Approved for public release; distribution unlimited.

DTIC
ELECTE
AUG 16 1984

B

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base, NM 87117

DTIC FILE COPY

84 08 14 012

This final report was prepared by LuTech, Inc, Hayward California, under Contract F29601-82-C-0027, Job Order 37630131 with the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico. Mr Carl E. Baum (NTAAT) was the Laboratory Project Officer-in-Charge.

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely Government-related procurement, the United States Government incurs no responsibility or any obligation whatsoever. The fact that the Government may have formulated or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication, or otherwise in any manner construed, as licensing the holder, or any other person or corporation; or as conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been authored by a contractor of the United States Government. Accordingly, the United States Government retains a nonexclusive, royalty-free license to publish or reproduce the material contained herein, or allow others to do so, for the United States Government purposes.

This report has been reviewed by the Public Affairs Office and is releasable to the National Technical Information Services (NTIS). At NTIS, it will be available to the general public, including foreign nations.

If your address has changed, if you wish to be removed from our mailing list, or if your organization no longer employs the addressee, please notify AFWL/NTAAT, Kirtland AFB, NM 87117 to help us maintain a current mailing list.

This technical report has been reviewed and is approved for publication.

Leonie D. Boehmer

LEONIE D. BOEHMER
Project Officer

Dan W Garrison

DAVID W. GARRISON
Lt Col, USAF
Chief, Applications Branch

FOR THE COMMANDER

Roger S. Case

ROGER S. CASE, JR
Lt Col, USAF
Chief, Aircraft & Missile Division

DO NOT RETURN COPIES OF THIS REPORT UNLESS CONTRACTUAL OBLIGATIONS OR NOTICE ON A SPECIFIC DOCUMENT REQUIRES THAT IT BE RETURNED.

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS			
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.			
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE						
4. PERFORMING ORGANIZATION REPORT NUMBER(S)			5. MONITORING ORGANIZATION REPORT NUMBER(S) AFWL-TR-84-21			
6a. NAME OF PERFORMING ORGANIZATION LuTech, Inc	6b. OFFICE SYMBOL (If applicable)		7a. NAME OF MONITORING ORGANIZATION Air Force Weapons Laboratory			
6c. ADDRESS (City, State and ZIP Code) 3516 Breakwater Court Hayward CA 94545			7b. ADDRESS (City, State and ZIP Code) Kirtland Air Force Base NM 87117			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F29601-82-C-0027			
8c. ADDRESS (City, State and ZIP Code)			10. SOURCE OF FUNDING NOS.			
			PROGRAM ELEMENT NO. 64711F	PROJECT NO. 3763	TASK NO. 01	WORK UNIT NO. 31

11. TITLE (Include Security Classification)
ALTERNATIVE LABELLING SCHEMES IN ELECTROMAGNETIC TOPOLOGY

12. PERSONAL AUTHOR(S)
Noss, R. S.

13a. TYPE OF REPORT Final Report	13b. TIME COVERED FROM Oct 82 TO Oct 83	14. DATE OF REPORT (Yr., Mo., Day) 1983, June	15. PAGE COUNT 28

16. SUPPLEMENTARY NOTATION
Work performed under Subcontract No. DC-SC-1026-4 for Dikewood, Division of Kaman
Sciences Corporation.

17.	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Electromagnetic Topology, Graph Theory, Interaction
12	01		Sequence Diagrams, Labelling Schemes, Applications

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
This report defines the problems associated with applying graph theory to interaction
sequence diagrams used in electromagnetic topology. Because few applications have been
developed, the optimal labelling scheme has yet to emerge. As a result, operations on
the graph are difficult to record concisely. This report contains criteria to guide
the design of labelling schemes. Some specific schemes are presented and illustrated
for a sample topology, and satisfaction of the criteria is discussed.

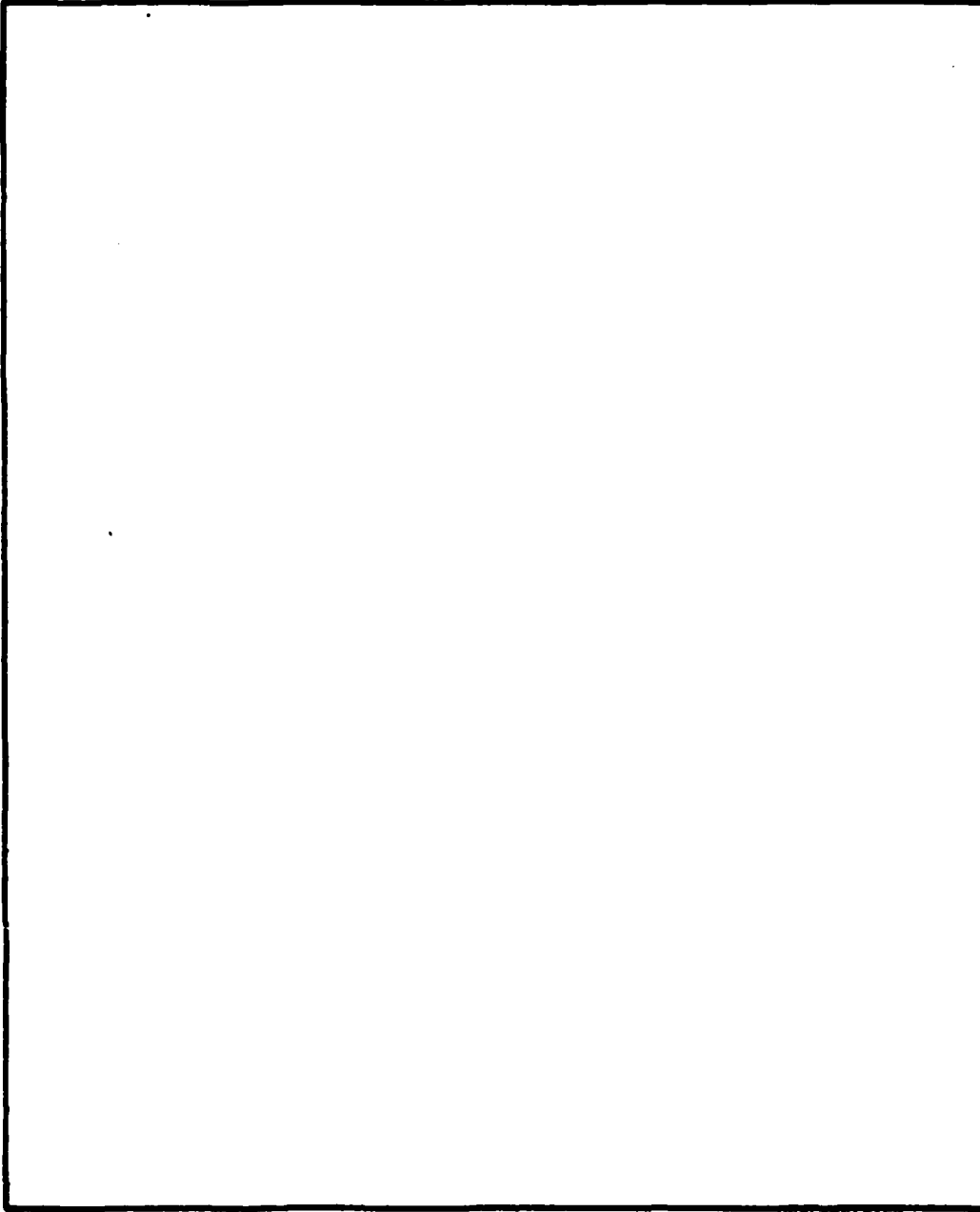| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION	
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Carl E. Baum	22b. TELEPHONE NUMBER (Include Area Code) (505) 844-0326	22c. OFFICE SYMBOL NTAAT

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.

CONTENTS

FIGURES

I. INTRODUCTION

Although electromagnetic topology (EMT) is a new concept, its theoretical development has already become overburdened with indices. References 1 through 3 describe the subdivision of Euclidean space into components of interest, namely shields and subshields (surfaces), layers and sublayers (volumes), etc., and suggests one method of labelling them. The same labels apply to the dual graph, in which volumes are represented by points, and surfaces between volumes as lines connecting the points. In this scheme, a given hierarchiacal level is identified with a volume called a layer. A layer is subscripted once to identify its hierarchical level. Nonintersecting subsets of a layer, called sublayers, receive two indices. The first index is the layer index. All of a layer's sublayers are numbered consecutively to supply the second integer of the pair.

Further decomposition of sublayers into subsets called elementary volumes follows the same pattern with a third subscript. Shields and subshields are tagged similary, but require two such sets of indices to show enclosed and enclosing volumes. A sample EMT, to sublayer level, is shown in Figure 1 along with its dual graph in Figure 2, labelled according to the scheme described.

Rather than dealing with these previously used labels, this report defines some characteristics of a well-labelled dual graph and presents a few alternative schemes which possess some of these traits.

* LAYER V_1 (= sublayer $V_{1,1}$)
(external source)

shield $S_{1,2}$

subshield $S_{2,1;3,1}$

sublayer $V_{3,1}$

subshield $S_{3,2;4,1}$

LAYER V_2
(= sublayer $V_{2,1}$)

sublayer $V_{4,1}$

subshield $S_{2,1;3,2}$

sublayer $V_{3,2}$

sublayer $V_{4,2}$
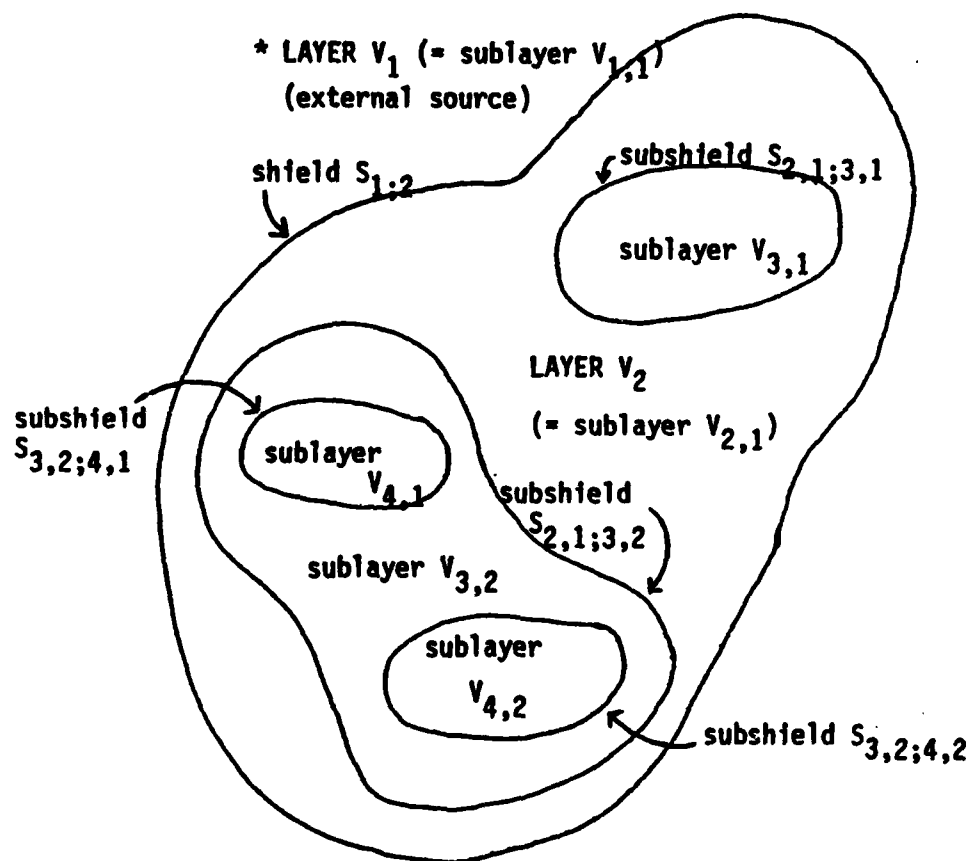
subshield $S_{3,2;4,2}$

Figure 1. A sample electromagnetic topology to sublayer level.
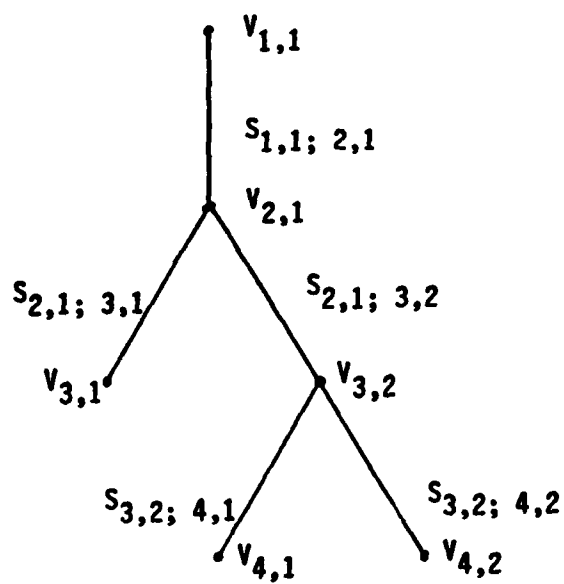
4

Figure 2. Dual graph of Figure 1.

## II. CHARACTERISTICS OF AN IDEAL LABELLING SCHEME

An ideal labelling scheme should meet the following criteria:

(1) Labels should be compatible with standard terminology. Representing an EMT by its dual graph invites the introduction of results from graph theory. For ease of application, the EMT graph labels must be clearly related to, if not the same as, standard graph theory notation. This requires identification of each vertex and edge, and adherence to the concepts of adjacency and nonadjacency of edges and vertices. (Two vertices are adjacent if they are the end points of some edge. Two edges are adjacent if some vertex is an end point of both.)

(2) The dual graph is said to be *a directed graph* (digraph) if each edge is oriented by distinguishing a starting vertex from an ending vertex. A digraph is hierarchical if there exists a partial ordering of the vertices. A tree is a graph containing no closed loops (cycles). Figure 3 illustrates these terms. Labels should indicate direction for digraphs, and ordering for hierarchical graphs, but if no ordering is assumed then the additional structure imposed by this requirement should be removable. That is, the label attached to a point of the graph should indicate which layer(s) enclose and are enclosed by it, as well as which layers are at the same hierarchical level (if the graph is hierarchical). Labels for vertices which neither precede nor follow each other should reflect this fact.
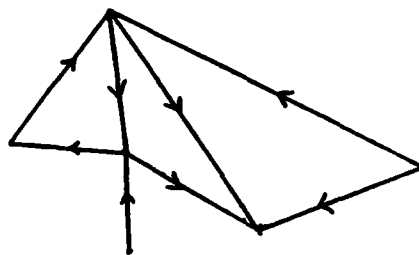
(a) directed graph.
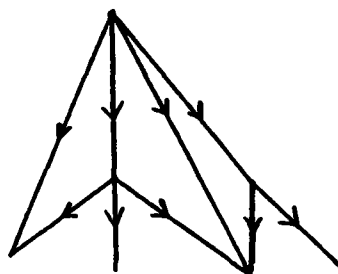
(b) hierarchical graph
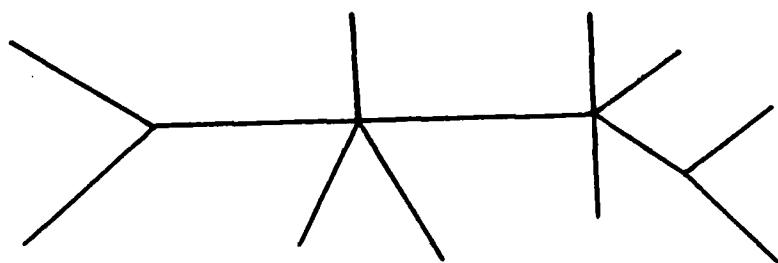
(c) tree graph.

Figure 3. A directed graph, a hierarchical graph, and
a tree graph.

(3) Labels should be flexible enough to absorb changes in the EMT, including

>   (a)   insertion of a subshield or sublayer.
>
>   (b)   deletion of a subshield or sublayer,
>
>   (c)   linking a sublayers (by a wire, hole, etc.),
>
>   (d)   frequency-dependent changes,

and   (e)   distinction between inside and outside of subshields.

Flexibility is the most stringent requirement, because the classification of a graph as a tree, digraph, hierarchical graph, etc., is changed when the topology is altered.

(4) Labels contain information about the distance between two vertices. In References 1 through 3 the term relative shielding order is used for the number of intervening subshields between two sublayers. Thus labels should generate the distance matrix, $D = (d_{ij})$, where d_{ij} might be defined as the minimum number of edges between vertices i and j , or as some generalization which satisfies the conditions of a metric.

(5) Labels should admit an operation representing inversion with respect to a pivot layer or pivotal sublayer. Inversion is a distance-preserving operation which uses reciprocity to consider the pivotal sublayer as the source of electromagnetic excitation. The operation may be visualized as reaching inside the topological diagram and pulling the pivotal sublayer to the outside, turning part of the diagram inside-out in the process. The corresponding operation on the dual graph consists of choosing a new base point, or suspending the graph by the pivotal vertex. If the graph is hierarchical, this raises the pivotal vertex to the top of the hierarchy and

partitions the vertices into two subsets, those for which the hierarchy
is preserved and those for which the hierarchy is reversed. The latter is
an order set, and the new label set can be generated by reversing the order
on that set.

(6) Labels should suggest how to apply concepts from relevant fields
of mathematics, including graph theory, abstract algebra, algebraic topology,
matrix theory, differential geometry, and analysis.

This is a lot to ask of a labelling scheme. A reasonable compromise
is to allow two or more schemes to collectively satisfy these requirements.
The schemes presented in the following section have been designed to satisfy
Criteria 2, 4 and 5 for hierarchical graphs, while still observing the
guidelines 1 & 6.

9

III. ALTERNATIVE LABELLING SCHEMES

The emphasis of this report is on hierarchical tree digraphs, so some of the considerations in Criterion 3 are not addressed. The following notation will be used throughout: Unless otherwise specified, X is a partially ordered set which may be represented as a tree graph. The $V(X)$, or just V if the set X is understood, is the set of labels representing the vertices of X, called the vertex set of X. Similarly, $E(X)$, or just E, is the edge set of X. A partial ordering $f<g$ is read "f precedes g" or "g follows f". Several schemes attach more than one character to a vertex. One of these should remain fixed during inversion, and is referred to as the identifier of of the vertex.

In graph theory, the standard way of labelling a graph is by specifying the vertex set and edge set. If there are p vertices, then there will be at most $q = \binom{p}{2} = p(p-1)/2$ edges. If the graph is a tree, then $q = p-1$. The vertices are arbitrarily labelled from the vertex set

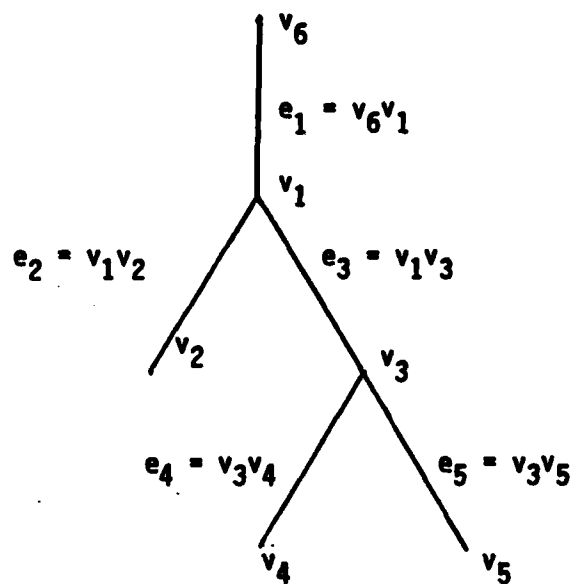$$V = \{v_1, v_2, \ldots, v_p\},$$

and the edge set is defined by

$$E = \{e_1, e_2, \ldots, e_q\}$$
$$= \{v_{i_1 j_1}, v_{i_2 j_2}, \ldots, v_{i_q j_q}\},$$

where vertex v_{i_k} is connected by edge e_k to vertex v_{j_k}. Figure 4 shows the standard labels applied to Figure 2, as well as the sets V and E. The order of v_{i_k} and v_{j_k} is important for a digraph and is implicitly

$$V(X) = \left\{ v_1, v_2, v_3, v_4, v_5, v_6 \right\}$$

$$E(X) = \left\{ e_1, e_2, e_3, e_4, e_5 \right\}$$

$$= \left\{ v_6 v_1, v_1 v_2, v_1 v_3, v_3 v_4, v_3 v_5 \right\}$$

Figure 4. Standard labelling scheme, vertex set $V(X)$, and edge set $E(X)$.

11

assumed to be fixed. There are cases in which this assumption hinders the analysis: Figure 5a shows a partially-directed graph possessing some directed edges and some undirected edges, requiring the introduction of extra edges (Fig. 5b), which create artificial cycles in the graph. The acceptability of this depends on the application of the graph.

If X is partially ordered (and thus has the property that no vertex v_k appears twice in the second position of the elements of the edge set), then inspection of the elements of E will reveal a unique vertex v_i which appears in the first position of a pair but not in the second position of any other pair. From this fact the tree can be reconstructed and thus admits an inversion operation. This method is sufficiently general to satisfy Criteria 1, 2, and 6, but distances cannot be conveniently calculated without first reconstructing the graph.

Another scheme of attaching labels to a graph containing p points is to select elements of a group G generated by $p-1$ elements, $v_1, v_2, \ldots, v_{p-1}$, which satisfy the relations

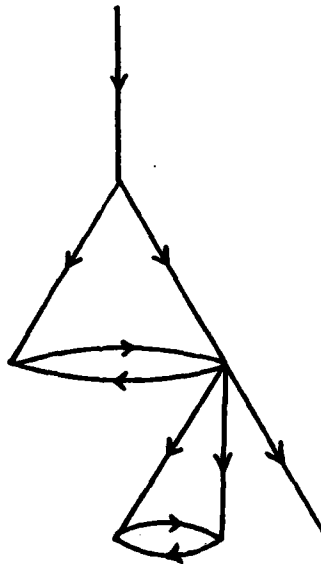(1) $v_i^2 = 1$ $\quad$ ($i = 1 \ldots, p-1$)

and

(2) $v_i v_j = v_j v_i$ $\quad$ ($i,j = 1, \ldots, p-1$)

The set (1) identifies G with p copies of the cyclic group Z_2. The second set abelianizes G. This scheme will be named the pZ_2 scheme.
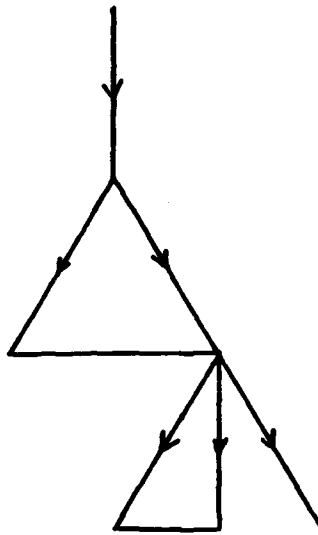
(a) A partially-directed graph.

(b) Extra edges introduced.

Figure 5. The problems of partially-directed graphs.

13

To understand the concept of groups, generators, and relations, consider a group G generated by two elements, a and b. This is a free group (because it is free from relations) containing infinitely many elements, including

1, a, a^{-1}, b, b^{-1}, a^2, ab, ba, aba, $ba^{-1}b^3a^{-2}b$, etc

For this example, the relation (1) becomes:

(1a) $a^2 = 1$ or $a = a^{-1}$

and

(1b) $b^2 = 1$ or $b = b^{-1}$

Including relations (1a) and (1b) reduces the elements of G to the form

1, a, b, ab, ba, aba, bab, abab, baba, etc

The group still has infinitely many elements, however.

Applying the relation (2) to the generators a and b yields:

(2)   ab = ba

Now G is a finite group:

G = {1, a, b, ab}

14

The element abab, for instance, has been reduced to abba by relation (2),
then further reduced to 1 by relations (1a) and (1b).

For the EMT in Figure 1, the five generators v_1, v_2, ..., v_5 yield
a group G containing 16 elements:

$$G = \{1, v_1, v_2, v_3, v_4, v_5, v_1v_2, v_1v_3, v_1v_4,$$
$$v_1v_5, v_2v_3, v_2v_4, v_2v_5, v_3v_4, v_3v_5, v_4v_5\}$$

Figure 6 illustrates the selection of V(X) from G. The identity
element of G is assigned to the unbounded region of Figure 1, representing
the source of electromagnetic excitation. This region corresponds to the
base point at the top of the tree. There is one layer containing all sub-
layers in the EMT. The corresponding point is labelled by v_1, a generator
of G. Two sublayers enclosed by this layer are identified with the group
elements v_1v_2 and v_1v_3, showing their relationship to v_1. Finally,
elements $v_1v_3v_4$ and $v_1v_3v_5$ are selected from G to represent the two
elementary volumes within v_1v_3, again showing the hierarchical dependence.
This scheme may be continued by assigning a generator of G multiplied by
the generator of every preceding vertex.

Criterion 1 is not easily satisfied (without first reconstructing the
graph from the sets V and E, which takes time) unless the Abelianization
relations are removed. In that case, X can be identified with the standard
vertex set V by keeping only the rightmost multiplicands of each element,
and writing v_p for the source element 1. The edge set E may be obtained
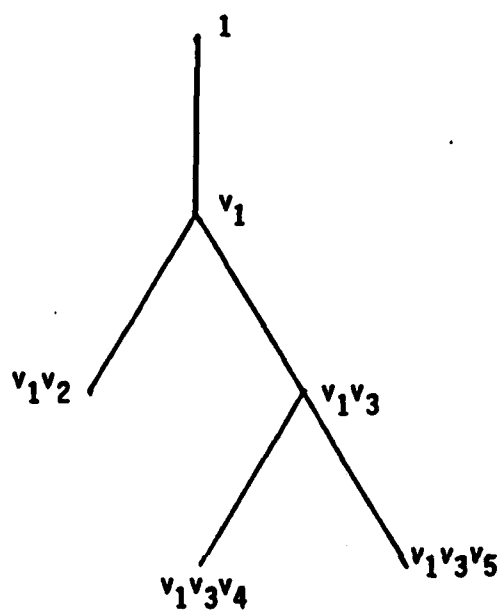by keeping the two rightmost multiplicands of each element, remembering to

Figure 6. The pZ_2 labelling scheme.

write an edge $v_p v_i$ for each element v_i which consists of only one generator. Thus applications of graph theory may be incorporated into this scheme.

By their selection from the group G, the labels clearly exhibit the hierarchical structure of the EMT: layer f precedes layer g if and only if every generator of f is also present in g (f and g are assumed to be reduced to simplest form by application of the set of relations 1). The distance between layers f and g is the number of generators in their product (reduced). This generates the distance matrix D. The multiplication table of the set X and the matrix D are shown in Table 1.

Inversion with respect to a pivot layer f is accomplished by multiplying the label of each vertes by f^{-1} (note that f^{-1} = f since G is Abelian), then reducing. This operation preserves distances, but the distance matrix is changed by a similarity transformation.

Figure 7 shows the result of inverting with respect to vertex $v_1 v_3$. The original labels are used in Figure 7a, then multiplied by $v_1 v_3$ and reduced to yield Figure 7b. Note that the order of the generators is ambiguous because G is Abelian. Table 2 reveals that the distance matrix for the postinversion graph is related to the matrix D for the preinversion graph by a similarity transformation $D \rightarrow A^{-1} D A$, where A is the 6 x 6 identity matrix I_6 with rows rearranged according to the permutation (316), so A^{-1} is I_6 with columns permuted by (316). (The permutation [316] applied to the sequence [1,2,3,4,5,6] produces the sequence [3,2,6,4,5,1].)

17

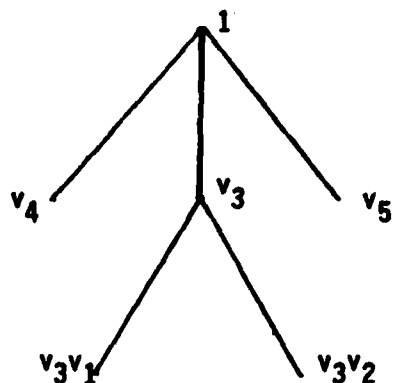Table 1. MULTIPLICATION TABLE AND DISTANCE MATRIX FOR FIGURE 6

	v_1	v_1v_2	v_1v_3	$v_1v_3v_4$	$v_1v_3v_5$	1
v_1	1	v_2	v_3	v_3v_4	v_3v_5	v_1
v_1v_2	v_2	1	v_2v_3	$v_2v_3v_4$	$v_2v_3v_5$	v_1v_2
v_1v_3	v_3	v_2v_3	1	v_4	v_5	v_1v_3
$v_1v_3v_4$	v_3v_4	$v_2v_3v_4$	v_4	1	v_4v_5	$v_1v_3v_4$
$v_1v_3v_5$	v_3v_5	$v_2v_3v_5$	v_5	v_4v_5	1	$v_1v_3v_5$
1	v_1	v_1v_2	v_1v_3	$v_1v_3v_4$	$v_1v_3v_5$	1

Distance Matrix for Figure 6

$$D = \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 1 \\ 1 & 0 & 2 & 3 & 3 & 2 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 0 & 2 & 3 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix}$$

18
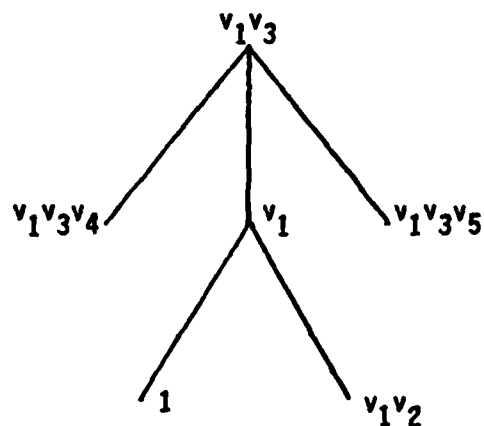
(a) Before relabelling.

(b) After relabelling by group multiplication.

Figure 7. Inversion of electromagnetic topology of Figure 6 with respect to vertex v_1v_3, using the pZ_2 scheme.

19

Table 2. TRANSFORMATION OF DISTANCE MATRIX FOR FIGURES 6 AND 7

$$\text{Before: } D = \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 1 \\ 1 & 0 & 2 & 3 & 3 & 2 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 0 & 2 & 3 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix}$$

After:

$$A^{-1}DA = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 1 \\ 1 & 0 & 2 & 3 & 3 & 2 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 1 & 3 & 3 & 2 \\ 2 & 0 & 1 & 3 & 3 & 2 \\ 1 & 1 & 0 & 2 & 2 & 1 \\ 3 & 3 & 2 & 0 & 2 & 1 \\ 3 & 3 & 2 & 2 & 0 & 1 \\ 2 & 2 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The set X is partitioned into three subsets relative to a pivot
element f:

   (a) $X_<(f)$ = {elements which precede f}

   (b) $X_>(f)$ = {elements which follow f}

   (c) $X_0(f)$ = {elements which neither precede nor follow f}

After inverting with respect to f the sets $X_<$ and X_0 vanish, and
$X = X_>$. As can be observed from the example, the permutation of I_6 reverses
the identifiers of elements in $X_<$ while leaving fixed the identifiers in
$X_>$ and X_0. Thus, although this scheme fails Criterion 5, there is an algor-
ithm to recover the original identifiers of the vertices.

A variation of the pZ_2 scheme removes both sets of relation, and
redefines inversion with respect to a pivot element f to be $f^{-1}g$. The
same remarks apply concerning satisfaction of the criteria, because the two
schemes are essentially the same: the pZ_2 scheme defined $f = f^{-1}$ to all
multiplication by f instead of f^{-1} in the inversion operation. The only
difference is now G is a free group on p-1 generators, a much larger
group. This free group scheme has the advantage that the ordering of gener-
ators in each label is correct after each inversion, whereas multiplication
in the pZ_2 scheme changes the order of some labels, so that closer inspec-
tion is required to identify predecessors.

The natural correspondence between the group G given in the pZ_2
scheme and the corners of the p-cube defines an embedding of the graph X
along certain edges of the cube. A corner is identified by an ordered

p-tuple $(b_1, \ldots, b_p)$, where each b_i is either 0 or 1. A label f assigns values to the b_i's as follows:

$$b_i = \begin{cases} 1 & \text{if } v_i \text{ is present in the label } f \\ 0 & \text{if } v_i \text{ is not present.} \end{cases}$$

Here the identity element 1 (base point of the tree) has been changed to v_p. If the cube is suspended by the base point, vertical height corresponds to hierarchical level, and inversion corresponds to suspending the cube by the pivot vertex. Although simple to describe, this embedding is not very useful, because the 3-cube allows graphs with only three vertices, and higher-dimensional cubes are not easy to visualize.

CONCLUSION

Graph theory plays an important role in describing electromagnetic topology. To make full use of the concepts of the theory, a systematic method of labelling graphs must be used. In this report some criteria were developed to aid in the design of labelling schemes. The criteria for labels were: (1) should be compatible with standard terminology; (2) should indicate direction for digraphs; (3) should be flexible enough to absorb changes in EMT; (4) should generate the distance matrix; and (5) should admit an operation representing inversion with respect to a pivot layer or a pivot sublayer. Using these criteria, various methods of labelling graphs were introduced. Satisfaction of the criteria was discussed. Work remains to be done, particularly in the area of flexibility of the labels.

REFERENCES

1. Baum, C.E., Electromagnetic Topology: A Formal Approach to the Analysis and Design of Complex Electronic Systems, <u>Interaction Note 400</u>, September 1980.

2. Baum, C.E., Sublayer Sets and Relative Shielding Order in Electromagnetic Topology, <u>Interaction Note 416</u>, April 1982.

3. Baum, C.E., Topological Considerations for Low-Frequency Shielding and Grounding, <u>Interaction Note 417</u>, June 1982.

4. Behzad and Chartrand, <u>Introduction to the Theory of Graphs</u>, Allyn & Bacon, Inc., Boston, 1971.

5. Massey, <u>Algebraic Topology: An Introduction</u>, Springer-Verlag, New York, 1967.

END

FILMED

9-84

DTIC